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Functional Form in Finished Goods Inventory Investment

A Note by Michael A. S. Guth

Empirical evidence continues to suggest that changes in inventory stock levels are a source of fluctuations in the gross national product. Maccini and Rossana (1984) have employed the inventory literature's leading model of finished goods investment: a multivariate flexible accelerator equation with a log-linear functional form. Maintained hypotheses about functional form should be tested, especially if they are hypotheses that permeate the empirical literature in an area. We propose to test the log-linear form so widely used in inventory demand equations.

Because inventory data is based on time series, functional form misspecification can compound the problems of inference with serially correlated errors. The present paper aims to simultaneously test the Maccini and Rossana model for functional form and autocorrelation as suggested by Savin and White (1978).

The inventory investment model can easily be adapted to the Savin-White procedure by including a lagged dependent variable as one of the regressors on which the Box-Cox transformation is applied. The extended model is then fitted to inventory data from U.S. manufacturing to test for functional form and autocorrelated errors. The results of our analysis are then compared with those of Maccini and Rossana (1984) who worked with the same data.

The traditional flexible accelerator or partial adjustment equation assumes firms adjust inventories proportional to the difference between desired and actual levels of given stocks. For the dynamic process we will consider, the partial adjustment model is:

$$\Delta H_t = \Phi(H^*_t - H_{t-1}) \, , \tag{1}$$

where $0 < \Phi \leqslant 1$, Δ is the first difference operator, H_t is the firm's actual finished goods inventory stock in period t, $H_t{}^*$ is the firm's desired inventory stock,

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and Φ denotes the fractional adjustment between the firm's desired and actual inventory holdings during a given period. Firms determine their desired inventory level H_t^* from their forecasts of the demand and cost variables such as the normal level of industry orders, expected money wage rate, expected raw materials prices, expected interest rate, and the industry's normal prevailing price level.

Previous efforts to model firms' inventory adjustments have often yielded relatively low adjustment speed coefficients of less than 10 percent per quarter. These findings would imply $\Phi = 0.10$, and firms increase their actual inventory levels to only 10 percent of their desired levels. Feldstein and Auerbach (1976) found these estimates implausible since most firms can effect wide swings in inventory levels through only a few days' production. In contrast, Maccini and Rossana derived adjustment speeds ranging between approximately 65 and 90 percent. Maccini and Rossana attributed their results to appropriate statistical methods: use of Hatanaka's (1974) serial correlation correction for lagged dependent variable models.

Their multivariate partial adjustment model is

$$\ln H_t = \alpha_0 + \sum_{i=1}^{L_1} \alpha_i^1 \ln Q_{t-i} + \sum_{i=1}^{L_2} \alpha_i^2 \ln \left(\frac{V}{P}\right)_{t-i} + \sum_{i=1}^{L_3} \alpha_i^3 \ln \left(\frac{W}{P}\right)_{t-i}$$
$$+ \sum_{i=1}^{L_4} \alpha_i^4 R_{t-i} + (1 - \Phi) \ln H_{t-1} - \delta \ln U_{t-1} - \eta \ln G_{t-1}$$
$$- \theta \ln M_{t-1} - \mu \ln E_{t-1} - \varphi \ln Y_{t-1} + u_t \,. \quad (2)$$

The Greek letters denote parameters to be estimated, and the variables in equation (2) are defined in Table 1. Maccini and Rossana (1984) discuss their data sources and additional reasons for selecting the variables in (2).

TABLE 1

LIST OF VARIABLES

Symbol	Variable Name
H_t	Finished goods inventory stock
P_t	Producer Price Index
Q_t	Flow of deflated nominal new orders
V_t	Commodity price index for crude materials
R_t	The ex post real interest rate for 4–6 month commercial rate
W_t	Production and nonsupervisory laborers' average hourly earnings
U_t	Unfilled orders stock deflated by P_t
G_t	Goods-in-process inventories
M_t	Stock of materials and supplies inventories
E_t	Stock of production and nonsupervisory workers
Y_t	Industrial production index
u_t	Random variable capturing unplanned inventory investment

Equation (2) must be recast into the generalized functional form in order to test for both autocorrelation and a log-linear specification:

$$H_t^{(\lambda)} = \alpha_0 + \sum_{i=1}^{L_1} \alpha_i^1 Q_{t-i}^{(\lambda)} + \sum_{i=1}^{L_2} \alpha_i^2 \left(\frac{V}{P}\right)_{t-i}^{(\lambda)} + \sum_{i=1}^{L_3} \alpha_i^3 \left(\frac{W}{P}\right)_{t-i}^{(\lambda)}$$
$$+ \sum_{i=1}^{L_4} \alpha_i^4 R_{t-i}^{(\lambda)} + (1 - \Phi) H_{t-1}^{(\lambda)} - \delta U_{t-1}^{(\lambda)} - \eta G_{t-1}^{(\lambda)}$$
$$- \theta M_{t-1}^{(\lambda)} - \mu E_{t-1}^{(\lambda)} - \varphi Y_{t-1}^{(\lambda)} + u_t \quad (3)$$

where λ denotes the Box-Cox functional form coefficient. Recall that the log-linear functional form is only a special case of (3) where $\lambda = 0$; linear functional form is a special case of (3) where $\lambda = 1$.

Let ρ denote the first-order autocorrelation parameter so that

$$u_t = \rho u_{t-1} + \epsilon_t ,$$

with $|\rho| < 1$, and the ϵ_t are assumed to be independent and $N(0,\sigma^2)$ random variables. We can test equation (3) for a log-linear functional form assuming the errors have no serial correlation. The conditional hypothesis becomes H_0: $\lambda = 0$ given $\rho = 0$; the alternative is H_1: $\lambda \neq 0$ given $\rho = 0$. We can then perform a large sample likelihood ratio (LR) test of the conditional hypothesis for significance for significance level α expressed by

$$C(\lambda) = 2[L(\hat{\lambda},\rho = 0) - L(\lambda = 0,\rho = 0)] > \chi_\alpha^2(1). \quad (4)$$

We could also use a large sample LR test for the more general hypothesis H_0: $\lambda = 0$ against H_1: $\lambda \neq 0$. At level α, we see the test once again has critical region

$$G(\lambda) = 2[L(\hat{\lambda},\hat{\rho}) - L(\lambda = 0,\hat{\rho})] > \chi_\alpha^2(1) . \quad (5)$$

In preliminary work with their time series data, Maccini and Rossana tested for an AR(1) error term process given their log-linear functional form. Their conditional hypothesis was H_0: $\rho = 0$ given $\lambda = 0$ with the alternative H_1: $\rho \neq 0$ given $\lambda = 0$. The conditional large sample LR test at significance level α would be

$$C(\rho) = 2[L(\lambda = 0,\hat{\rho}) - L(\lambda = 0,\rho = 0)] > \chi_\alpha^2(1) . \quad (6)$$

More generally, Maccini and Rossana could have tested for autocorrelated error terms without the log-linear functional form constraint. The general hypothesis H_0: $\rho = 0$ against H_1: $\rho \neq 0$ at level α suggests a critical region for

$$G(\rho) = 2[L(\hat{\lambda},\hat{\rho}) - L(\hat{\lambda}, \rho = 0)] > \chi_\alpha^2(1) . \quad (7)$$

Finally, we can jointly test that the errors have no serial correlation, and the model fits a log-linear functional form. The joint test has null hypothesis H_0: $\lambda = 0, \rho = 0$ with the alternative $H_1: \lambda \neq 0$ and/or $\rho \neq 0$ with critical region

$$J(\lambda, \rho) = 2[L(\hat{\lambda}, \hat{\rho}) - L(\lambda = 0, \rho = 0)] > \chi^2_\alpha(2) . \tag{8}$$

The regression coefficients for each equation apply to the transformed variables, not the original variables. Thus, we cannot meaningfully compare the estimated coefficients for models with different λ values, i.e., models with different functional form. However, we can compare the different models' estimated elasticities. For the equation with Box-Cox transformations on both the dependent and explanatory variables, a given regressor's elasticity evaluated at the sample mean is

$$E_k = \frac{\partial Y}{\partial X_k} \frac{\bar{X}_k}{\bar{Y}} = \hat{\beta}_k \frac{\bar{X}_k^\lambda}{\bar{Y}^\lambda} .$$

The data used in this analysis was divided into durable, nondurable, and total manufacturing sectors. With the total manufacturing data, we first estimated the log-linear model, i.e., $\lambda = 0$, specifying no AR(1) error term hypothesis. The second model then kept the log-linear functional form constraint, but iterated a $\hat{\rho}$ value of 0.05 to maximize the log-likelihood function. Third, the Box-Cox Extended (BCE) model derived a λ estimate of 0.55. Finally, we simultaneously tested for functional form and autocorrelation in the unrestricted Box-Cox Extended Autoregressive (BCEA) model.

TABLE 2

TOTAL MANUFACTURING DATA 1957–1981[1]

Model	λ	ρ	$L(\lambda, \rho)$	$E_{H_{t-1}}$
LOG	0	0	2549.7980	0.97242
LOG AUTO	0	0.05	2550.0713	0.96847
BCE	0.55	0	2555.5093	0.97746
BCEA	1.05	0.16	2555.5537	0.95134

Test	$C(\lambda)$	$G(\lambda)$	$C(\rho)$	$G(\rho)$	$J(\lambda, \rho)$
value	11.4226	10.9648	0.5466	0.0888	11.5114

[1] To conform to Maccini and Rossana's specification we set $\eta = 0$. The $\frac{W}{P_{t-i}}$ and R_{t-i} variables were omitted, and lag lengths chosen as follows: $L_1 = 36$, $L_2 = 9$. The empirical estimates were calculated on White's (1978) econometrics package SHAZAM.

The BCEA model allows the data to fit both the functional form and autocorrelation parameters simultaneously. For the total manufacturing data the optimal λ and ρ are 1.05 and 0.16, respectively. From the $C(\lambda)$ test, we reject H_0: $\lambda = 0$, given $\rho = 0$ in favor of $H_1: \lambda \neq 0, \rho = 0$. Moreover, we also reject $\lambda = 0$ by the $G(\lambda)$ test.

However, the autocorrelation parameter tests revealed the total manufacturing data exhibited less serial correlation than originally believed. We were unable to reject the hypothesis H_0: $\rho = 0$, given $\lambda = 0$, in favor of H_1: $\rho \neq 0$, given $\lambda = 0$, from the $C(\rho)$ test statistic. Furthermore, the $G(\rho)$ statistic proved we couldn't even reject H_0: $\rho = 0$ in favor of H_0: $\rho \neq 0$. Finally, the joint log-linear functional form/no AR(1) hypothesis H_0: $\lambda = 0$, $\rho = 0$ is rejected from the $J(\lambda,\rho)$ statistic. The lagged inventory elasticity only slightly differed among the models tested.

Tables 3 and 4 list the data for the durable and nondurable sectors, respectively. As easily seen from each of the large test values, the log-linear functional form and no AR(1) error terms hypotheses can always be rejected. However, the latter conclusion only applies for the general $G(\rho)$ statistic, as the $C(\rho)$ statistic in both sectors showed that a zero autocorrelation parameter test couldn't be rejected.

TABLE 3

DURABLE GOODS DATA 1957–1981[2]

Model		λ		ρ		$L(\lambda,\rho)$		$E_{H_{t-1}}$
LOG		0		0		3015.0255		0.98218
LOG AUTO		0		0.09		3015.9558		0.97791
BCE		0.80		0		3017.7722		0.95029
BCEA		1.00		0.40		3030.4562		0.94946
Test	$C(\lambda)$		$G(\lambda)$		$C(\rho)$		$G(\rho)$	$J(\lambda,\rho)$
value	5.4934		29.0008		1.8606		25.3680	30.8614

[2]cf. footnote 1, Table 1.

TABLE 4

NONDURABLE GOODS DATA 1957–1981[3]

Model		λ		ρ		$L(\lambda,\rho)$		$E_{H_{t-1}}$
LOG		0		0		3050.2164		0.93480
LOG AUTO		0		0.03		3050.2940		0.92849
BCE		0.90		0		3059.9262		0.95564
BCEA		1.00		−0.03		3074.9770		0.93823
Test	$C(\lambda)$		$G(\lambda)$		$C(\rho)$		$G(\rho)$	$J(\lambda,\rho)$
value	19.4196		49.3660		.1552		30.1016	49.5212

[3]Again Maccini and Rossana set η equal to 0, omitted the $\frac{W}{P_{t-i}}$ and R_{t-i} variables, and chose lag lengths of $L_1 = 30$, $L_2 = 12$.

Tse (1984) has suggested use of Lagrange multiplier tests for functional form instead of likelihood ratio tests. "Frequently there is no intention of adopting the general (unrestricted) Box-Cox transformed model even if both (linear and log-linear) restrictions on λ are rejected" (Tse 1984, p. 337). Our results for the BCEA model across all three sectors indicate the value of testing the unrestricted model. In each of these sectors the data best fit an approximate linear functional form.

For the data used in this study, we can reject the prevailing log-linear specification in favor of a linear model if $\lambda = 1.05$ is taken as an approximation of $\lambda = 1$.

We can conclude that for the data used in this study, a log-linear specification of inventory adjustment processes would be statistically incorrect. Our results also compare favorably with those of Feldstein and Auerbach (1976), who selected a linear inventory demand equation. However, we cannot conclude that the inventory literature should always employ linear adjustment models. Rather, our findings for an approximate linear functional form are limited to the data used in this analysis.

Functional form misspecification can lead to erroneous conclusions about the extent of serial correlation problems, as seen from the different $\hat{\rho}$ estimates and likelihood function values for the "LOG AUTO" and BCEA models. Additional discussion of inferences from models with functional form misspecification can be found in the handbook by Judge, Griffiths, Hill, and Lee (1980).

Finally, any economist working with inventory estimates is keenly aware of the major problems of measurement that the source data raise. Because the data used in this study was based on aggregates and contained different levels of rounding precision for the different variables, it is likely that errors in measurement or precision could affect statistical tests for functional form and autocorrelation. To the extent that our objective was to test the Maccini and Rossana (1984) model using the same data, we were limited in our efforts to correct for measurement errors.

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